A solution approach to minimum spanning tree problem under fermatean fuzzy environment

Francis Remigius Perpetua Mary[1], Swaminathan Mohanaselvi[2], Said Broumi[3]
[1]Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Chennai, India
[2]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco
[3]Regional Center for the Professions of Education and Training (C.R.M.E.F), Casablanca-Settat, Morocco

Article Info

Article history:

Received Sep 15, 2022
Revised Oct 24, 2022
Accepted Dec 23, 2022

Keywords:

Fermatean fuzzy number
Fuzzy minimum spanning tree
Improved score function
Optimum branching algorithm
Prim'salgorithm

ABSTRACT

In classical graph theory, the minimal spanning tree (MST) is a subgraph with no cycles that connects each vertex with minimum edge weights. Calculating minimum spanning tree of a graph has always been a common problem throughout ages. Fuzzy minimum spanning tree (FMST) is able to handle uncertainty existing in edge weights for a fuzzy graph which occurs in real world situations. In this article, we have studied the MST problem of a directed and undirected fuzzy graph whose edge weights are represented by fermatean fuzzy numbers (FFN). We focus on determining an algorithmic approach for solving fermatean fuzzy minimum spanning tree (FFMST) using the modified Prim's algorithm for an undirected graph and modified optimum branching algorithm for a directed graph under FFN environment. Since the proposed algorithm includes FFN ranking and arithmetic operations, we use FFNs improved scoring function to compare the weights of the edges of the graph. With the help of numerical examples, the solution technique for the proposed FFMST model is described.

Corresponding Author:

Swaminathan Mohanaselvi
Department of Mathematics, College of Engineering and Technology
SRM Institute of Science and Technology
Chennai–603 203, Tamilnadu, India
Email: mohanass@srmist.edu.in

1. INTRODUCTION

In conventional graph theory, minimal spanning tree (MST) [1] is a commonly used combinatorial optimization problem. Many researchers like Czech scientist Otakar Borůvka in 1926 was the first to propose the algorithm in finding the MST. The next version of the algorithm was by Prim [2] and Dijkstra in 1959 and further developed by Kruskal's [3]. Consequently, a lot of researchers diligently studied a lot to create an effective MST algorithm. Graham and Hell [4] explored the central role in the history of MST. The importance and popularity of the MST is due to real world applications such as to design network problems in transportation, telecommunication, and water supply. A MST's edge weights can be calculated using any arbitrary value assigned to the edges, including distance, traffic load, overcrowding, and so on. Uncertainty is not appropriately represented in classical graph theoryand hence almost all MST problem has the weights assigned to edges as real numbers but in relevant issues in everyday life the parameters have not naturally precise there exits unpredictability and ambiguity. To deal with the ambiguity and uncertainty, some researchers have utilized random variables to deal with the arc weight's lack of precision. This particular type of MST problem is characterized as stochastic minimum spanning tree (SMST) problem [5]-[7]. This assumption might not be true under realistic circumstances in the stochastic MST problem. They are practicable when the probability distribution function of the edge weight is assumed to be known, which is the main issue

with those SMST algorithms. Fuzzy set (FS) theory proposed by Zadeh [8] is a flexible and efficient tool for handling deals with imprecise and vague information. To properly articulate the imprecise information, Atanassov [9] put forth the intuitionistic fuzzy set (IFS) as an extension of FS by incorporating membership degree (m) with non-membership degree (n) of each element such that $0 \leq m + n \leq 1$. But if $m + n > 1$, then IFS is not anymore applicable. To overcome this Yager [10], [11] proposed pythogorean fuzzy set (PFS) with the condition$0 \leq m^2 + n^2 \leq 1$. Although PFS generalizes IFS, it cannot define the uncertanities when the condition fails and hence Senapati and Yager [12], proposed the fermatean fuzzy set (FFS) by enlarging the uncertainty domain with the condition $0 \leq m^3 + n^3 \leq 1$ and explored FFS properties. Rosenfeld [13] proposed the fuzzy analogue of numerous basic graph-theoretic conceptions and Bhattacharya [14], gave a note on fuzzy graphs (FG). Shannon and Atanassov [15], introduced the concept of intuitionistic fuzzy relations (IFR) and intuitionistic fuzzy graphs (IFG). Akram and Akmal [16], explored the IFG structure. Akram *et al*. [17] introduced pythagorean fuzzy graphs (PFG), an IFG of third type called fermatean fuzzy graph (FFG) and its application in decision making problems. MST problem of fuzzy nature involves edge weights as fuzzy numbers. A FMST problem traces its way back to 1996 and lasting till the recent years. Itoh and Ishii [18], proposed a fuzzy approach to the classical MST problem. Over the past two decades, many researchers have worked on FMST and have developed several algorithmic methods to solve it. Few of the approaches to FMST [18]-[26] are listed in Table 1.

Table 1. Existing approaches to FMST

S.No.	References	Authors	MST type	Method and discussions
1	[18]	Itoh *et al.*	FMST	Chance constrained programming based on necessity measure
2	[19]	Takahashi *et al.*	FMST	Genetic algorithm
3	[20]	Almeida *et al.*	FMST	Genetic algorithm
4	[21]	Janiak *et al.*	FMST	Possibility theory using fuzzy intervals
5	[22]	Verma *et al.*	FMST	Greedy algorithm using quasi-gaussian fuzzy weights
6	[23]	Dey *et al.*	FMST	Prim's algorithm in fuzzy environment
7	[24]	Zhou *et al.*	FMST	Fuzzy α–MST based on credibility measure
8	[25]	Dey *et al.*	FMST	Kruskal's algorithm
9	[26]	Deshpande *et al.*	FMST	Prim's, Kruskal's and Dijkstra's algorithms

Recently many researchers [27], [28] are exploring the developments of fuzzy sets in MST problems with different approaches. Real-world circumstances make it difficult for the decision maker (DM) to make precise determinations on the feasibilty of data pertinent to the system parameters. Let us consider a road network where the cities are represented by vertices and the edge weights indicate the costs associated with travelling between the cities. The cost of travelling depends on fuel price, driver's charge, and toll price each of which fluctuate from time to time and hence are uncertain in nature. Therefore, the edge weights are defined in a range rather than an exact number and can be defined by fermatean fuzzy numbers (FFNs). Compared to FS, IFS, and PyFS, the space of m, n for FFS is larger.

The comparison of existing approaches to extensions of FMST in Table 2 focuses on the research gaps and the prerequisite for this article. A thorough review of the prior FMST literature reveals that IFS and PFS environment are the only topics that are discussed. The purpose of this article is to incorporate MST into an FFN context. The main contribution of our proposed study is to solve the FFMST whose edge weights are FFNs by using the modified Prim's algorithm for an undirected graph and modified optimum branching algorithm for a directed graph. The following are the main contributions of the proposed work.

a. We have proposed an algorithmic approach for the FFMST problem for directed and undirected FFG.
b. For a directed FFG, an innovative concept of modified optimum branching algorithm is proposed for the first time in FFN environment.
c. For an undirected FFG, a modified Prim's algorithm is proposed in FFN environment.
d. The existing score function and arithmetic operations of FFNs are used in the proposed algorithm.
e. To prove the efficiency of the suggested modified Prim's algorithm, we have considered a problem of local bank that wants to build its network connecting its headquarters, branches and ATMs as an example for undirected FFG and obtained the lowest possible cost using the proposed Prim's algorithm.
f. Also, we have discussed another example for directed graph to explore the effectiveness of the proposed modified optimum branching algorithm.

The remaining part of this paper is structured as follows. We examine preliminary data in section 2, with respect to FFG, score function and arithmetic operations of FFN. A new algorithm is proposed in section 3, based on traditional Prim's algorithm and modified optimum branching algorithm is developed to determine FMST. In section 4, illustrative examplesare discussed. Finally, conclusion and future work of the proposed algorithms are deliberated in section 5.

Table 2. Existing approaches to extensions of FS in MST

S.No.	Author	Extensions of FS in MST	Graph type	Methodology
1	Mohanta *et al.* [29]	Intuitionistic FMST	Undirected graph	Bor˚uvka's algorithm
2	Dey *et al.* [30]	Interval type-2 FMST	Undirected graph	Genetic algorithm
3	Dan *et al.* [31]	Interval type-2 FMST	Undirected graph	Borüvka's algorithm
4	Habib *et al.* [32]	Pythagorean FMST	Directed graph	PF similarity measure
5	Proposed	Fermatean FMST	Directed graph	Prim's algorithm
6	Proposed	Fermatean FMST	Undirected graph	Optimum branching algorithm

2. PRELIMINARIES

This section, we have discussed about FG, IFG, FFG, score and arithmetic operations of FFNs to facilitate future discussion.

Definition 2.1:

A fuzzy subset $\mathbb{A}$ defined on a universal set $\mathbb{X}$ is a map $\mu_{\mathbb{A}}: \mathbb{X} \to [0,1]$. A fuzzy relation (FR) is a fuzzy subset $\mu_{\Re}: \mathbb{X} \times \mathbb{X} \to [0,1]$ on $\mathbb{X} \times \mathbb{X}$. That is:

$$\Re = \{\langle (x,y), \mu_{\Re}(x,y) \rangle | (x,y) \in \mathbb{X} \times \mathbb{X}\}$$

Definition 2.2:

A fuzzy graph$\mathcal{G} = (\mathbb{V}, \mathcal{V}, \mathcal{E})$ is a non-empty vertex set $\mathbb{V}$ along with pair of functions$\mathcal{V}: \mathbb{V} \to [0,1]$ and $\mathcal{E}: \mathbb{V} \times \mathbb{V} \to [0,1]$, such that, forevery $x, y \in \mathbb{V}$, $\mathcal{E}(x,y) \leq min\big(\mathcal{V}(x), \mathcal{V}(y)\big)$ where $\mathcal{V}(x)$ represents the membership degree of the vertex x and $\mathcal{E}(x,y)$ represents the membership degree of the edge (x,y).

Definition 2.3:

An IFR $\Re_{\mathcal{J}}$ is an IFS of the form:

$$\Re_{\mathcal{J}} = \left\{\langle (x,y), \mu_{\Re_{\mathcal{J}}}(x,y), \phi_{\Re_{\mathcal{J}}}(x,y) \rangle | (x,y) \in \mathbb{X} \times \mathbb{X}\right\}$$

where $\mu_{\Re_{\mathcal{J}}}: \mathbb{X} \times \mathbb{X} \to [0,1]$ and $\phi_{\Re_{\mathcal{J}}}: \mathbb{X} \times \mathbb{X} \to [0,1]$ and satisfies $0 \leq \mu_{\Re_{\mathcal{J}}}(x,y) + \phi_{\Re_{\mathcal{J}}}(x,y) \leq 1$.

Definition 2.4:

An IFG is a pair $\mathcal{G}_{\mathcal{J}} = (\mathbb{A}, \mathbb{B})$ where $\mathbb{A} = (\mathbb{V}, \mathcal{V}_{\mathbb{A}}, \mathcal{E}_{\mathbb{A}})$ is anIFS in $\mathbb{V}$ and $\mathbb{B} = (\mathbb{V} \times \mathbb{V}, \mathcal{V}_{\mathbb{B}}, \mathcal{E}_{\mathbb{B}})$ is an IFR on $\mathbb{V}$ such that:

$$\mathcal{V}_{\mathbb{B}}(x,y) \leq min\big(\mathcal{V}_{\mathbb{A}}(x), \mathcal{V}_{\mathbb{A}}(y)\big)$$

$$\mathcal{E}_{\mathbb{B}}(x,y) \leq min\big(\mathcal{E}_{\mathbb{A}}(x), \mathcal{E}_{\mathbb{A}}(y)\big)$$

Such that $0 \leq \mathcal{V}_{\mathbb{B}}(x,y) + \mathcal{E}_{\mathbb{B}}(x,y) \leq 1$ for all $x, y \in \mathbb{V}$.

Definition 2.5:

A Pythagorean fuzzy graph (PFG) is a pair $\mathcal{G}_{\mathcal{P}} = (\mathbb{A}, \mathbb{B})$ where $\mathbb{A} = (\mathbb{V}, \mathcal{V}_{\mathbb{A}}, \mathcal{E}_{\mathbb{A}})$ is an IFS in $\mathbb{V}$ and $\mathbb{B} = (\mathbb{V} \times \mathbb{V}, \mathcal{V}_{\mathbb{B}}, \mathcal{E}_{\mathbb{B}})$ is an IFR on $\mathbb{V}$ such that:

$$\mathcal{V}_{\mathbb{B}}(x,y) \leq min\big(\mathcal{V}_{\mathbb{A}}(x), \mathcal{V}_{\mathbb{A}}(y)\big)$$

$$\mathcal{E}_{\mathbb{B}}(x,y) \leq min\big(\mathcal{E}_{\mathbb{A}}(x), \mathcal{E}_{\mathbb{A}}(y)\big)$$

Such that $0 \leq \mathcal{V}_{\mathbb{B}}^2(x,y) + \mathcal{E}_{\mathbb{B}}^2(x,y)(x,y) \leq 1$ for all $x, y \in \mathbb{V}$.

Definition 2.6:

A Fermatean fuzzy set $\mathcal{F}$ defined on a universal set X is a structure having the form:

$$\mathcal{F} = \{\langle \alpha, \mathcal{V}_{\mathcal{F}}(\alpha), \mathcal{E}_{\mathcal{F}}(\alpha) \rangle | \ \alpha \in \mathrm{X}\}(2)$$

where $\mathcal{V}_{\mathcal{F}}: X \to [0,1]$ and $\mathcal{E}_{\mathcal{F}}: X \to [0,1]$ indicates the membership and the non-membership degree of$\alpha \in X$respectively and $0 \leq \mathcal{V}_{\mathcal{F}}^3(\alpha) + \mathcal{E}_{\mathcal{F}}^3(\alpha) \leq 1$ for all $\alpha \in X$.

For convenience, a FFN is denoted by$\mathcal{F} = (\mathcal{V}, \mathcal{E})$.

Definition 2.7:

A FFG is a pair $\mathcal{G}_{\mathcal{F}} = (\mathbb{A}, \mathbb{B})$ where $\mathbb{A} = (\mathbb{V}, \mathcal{V}_{\mathbb{A}}, \mathcal{E}_{\mathbb{A}})$ is an IFS in $\mathbb{V}$ and $\mathbb{B} = (\mathbb{V} \times \mathbb{V}, \mathcal{V}_{\mathbb{B}}, \mathcal{E}_{\mathbb{B}})$ is an IFR on $\mathbb{V}$ such that:

$$\mathcal{V}_{\mathbb{B}}(x,y) \leq min\big(\mathcal{V}_{\mathbb{A}}(x), \mathcal{V}_{\mathbb{A}}(y)\big)$$

$$\mathcal{E}_{\mathbb{B}}(x, y) \leq min\big(\mathcal{E}_{\mathbb{A}}(x), \mathcal{E}_{\mathbb{A}}(y)\big)$$

Such that $0 \leq \mathcal{V}_{\mathbb{B}}^3(x, y) + \mathcal{E}_{\mathbb{B}}^3(x, y)(x, y) \leq 1$ for all $x, y \in \mathbb{V}$.

Definition 2.8:

Given a fermatean digraph $\mathcal{G}_{\mathcal{F}} = (\mathbb{A}, \mathbb{B})$ and a vertex $r \in \mathbb{A}$, an arborescence (rooted at r) is a tree $\mathcal{T}_{\mathcal{F}}$ such that $\mathcal{T}_{\mathcal{F}}$ is a spanning tree of $\mathcal{G}_{\mathcal{F}}$ if we ignore the directions of edges. There is a directed unique path in $\mathcal{T}_{\mathcal{F}}$from $r$to each other node $\upsilon \in \mathbb{A}$.

The ranking of FFNs is an important issue in DM process as it is difficult to determine very clearly one FFN is larger or smaller than another due to its overlapping nature. The score function (SF) of FFN directly transforms each fuzzy number into a crisp real number and is used in the proposed algorithms for its simplicity of calculation. The existing arithmetic operations of FFN are also used in the proposed algorithms to perform addition of edge weights of FFMST.

Definition 2.9:

Let the set of all FFNs defined over the real line R be denoted by F(R). Let $\mathcal{F} = (\mathcal{V}, \mathcal{E})$, $\mathcal{F}_1 = (\mathcal{V}_1, \mathcal{E}_1)$ and $\mathcal{F}_2 = (\mathcal{V}_2, \mathcal{E}_2)$ be three FFNs and $\lambda > 0$. The basic arithmetic operation rules of FFNs are outlined:

$$\mathcal{F}_1 \oplus \mathcal{F}_2 = \langle \sqrt[3]{\mathcal{V}_1^3 + \mathcal{V}_2^3 - \mathcal{V}_1^3\mathcal{V}_2^3}, \mathcal{E}_1\mathcal{E}_2 \rangle$$

$$\mathcal{F}_1 \otimes \mathcal{F}_2 = \langle \mathcal{V}_1\mathcal{V}_2, \sqrt[3]{\mathcal{E}_1^3 + \mathcal{E}_2^3 - \mathcal{E}_1^3\mathcal{E}_2^3} \rangle$$

$$\lambda\mathcal{F} = \langle \sqrt[3]{1 - (1 - \mathcal{V}^3)^\lambda}, \mathcal{E}^\lambda \rangle$$

$$\mathcal{F}^\lambda = \langle \mathcal{V}^\lambda, \sqrt[3]{1 - (1 - \mathcal{E}^3)^\lambda}$$

Definition 2.10:

Sahoo [33], [34] proposed three different types of SFs of FFN and is applied to relate any two FFNs. Let $\mathcal{F} = (\mathcal{V}, \mathcal{E})$ be an FFN. Then the score function $SF_1(\mathcal{F})$, $SF_2(\mathcal{F})$ and $SF_3(\mathcal{F})$ of a FFN are defined as follows:

$$SF_1(\mathcal{F}) = \frac{1}{2}(1 + \mathcal{V}^3 - \mathcal{E}^3)$$

$$SF_2(\mathcal{F}) = \frac{1}{3}(1 + 2\mathcal{V}^3 - \mathcal{E}^3)$$

$$SF_3(\mathcal{F}) = \frac{1}{2}(1 + \mathcal{V}^2 - \mathcal{E}^2)|\mathcal{V} - \mathcal{E}|$$

Let $\mathcal{F}_1 = (\mathcal{V}_1, \mathcal{E}_1)$ and $\mathcal{F}_2 = (\mathcal{V}_2, \mathcal{E}_2)$ be any two FFNs and the score values are $SF_j(\mathcal{F}_i)$(i=1,2; j=1,2,3). Now any two FFNs can be ranked as described below:

If $SF_j(\mathcal{F}_1) < SF_j(\mathcal{F}_2)$, then $\mathcal{F}_1 < \mathcal{F}_2$;

If $SF_j(\mathcal{F}_1) > SF_j(\mathcal{F}_2)$, then $\mathcal{F}_1 > \mathcal{F}_2$;

If $SF_j(\mathcal{F}_1) = SF_j(\mathcal{F}_2)$, then $\mathcal{F}_1 = \mathcal{F}_2$;

3. PROPOSED METHOD

3.1. Fermatean fuzzy minimum spanning tree

A spanning tree is a subset of graph G, which has all the vertices covered with minimum possible number of edges. The spanning tree has no cycles and is connected. MST is the tree with the smallest possible length among all spanning trees. Uncertainty occurring in edge weights due to insufficient information is well handled by FS theory. Blue *et al*. [35] discussed about taxonomy of graph fuzziness and explored different variants of FG. In this article we have studied directed and undirected fermatean weighted connected graph whose edge weights are considered as FFNs. Also, we have proposed two algorithms to solve FFMST.

3.2. Modified optimum branching algorithm for directed graph FFMST

Consider a directed FFG. In order to minimise the total edge weights, the modified optimum branching algorithm identifies the subset of edge set $\mathbb{B}$ that contains all of the FFG's vertices. The following are the steps involved in modified optimum branching algorithm.

a. Input: directed FFG rooted at $r \in \mathbb{A}$
b. Step 1: determine the score function for the FFN.
c. Step 2: using the FFNs construct the FFG with their respective edge weights. It is important to ensure that the obtained edge weights are non - negative.
d. Step 3: initialize the root $r \in \mathbb{A}$ as the source.
e. Step 4: add the following vertex now, in any sequence.
f. Step 5: analyse the in degree of the chosen vertex and find the minimum weighted edge entering the chosen vertex. Rewrite the weights by using the difference of the minimum weight determined.
g. Step 6: repeat step 5 for all$v \neq a \in \mathbb{A}$.
h. Step 7: construct a subgraph $\mathcal{T}_{\mathcal{F}}$ of $\mathcal{G}_{\mathcal{J}}$ rooted at r such that $\mathcal{T}_{\mathcal{F}}$ has no directed cycles and each node $v \neq a$ has exactly one entering edge where $\mathcal{T}_{\mathcal{F}}$the required spanning tree is.
i. Step 8: determine all the possible graphs $\mathcal{T}_{\mathcal{F}}$, compute arborescence rooted at r of minimum cost.
j. Output: $\mathcal{T}_{\mathcal{F}}$, a FFMST for the given directed graph.

3.3. Modified Prim's algorithm for undirected FFMST

Consider an undirected FFG. In order to minimise the total edge weights, the modified Prim's algorithm identifies the subset of edge set $\mathbb{B}$ that contains all of the FFG's vertices. The following are the steps involved in modified Prim's algorithm.

a. Input: undirected FFG.
b. Step 1: choose an arbitrary start vertex from the vertex set $\mathbb{A}$ and denote it as root vertex.
c. Step 2: select an edge connecting the tree vertex and fringe vertex having the minimum edge weight. The minimum edge weight of FFMST can be calculated using the SF of FFNs.
d. Step 3: add the chosen edge to FFMST if it doesn't form any closed cycle.
e. Step 4: repeat steps 3 and 4 until the fringe vertices (vertices not included in FFMST) remain.
f. Output: $\mathcal{T}_{\mathcal{F}}$, a FFMST for the given undirected graph.

4. RESULTS AND EVALUATION

In this section we focus on illustrative examples emphasisizing the proposed modified optimum branching algorithm and modified Prim's algorithm. As FFMST have direct applications in the design of networks, we have considered two numerical examples for the directed and undirected fuzzy graphs.

4.1. Example 1

To determine the FFMST for the example given below by adopting the proposed modified optimum branching algorithm. Consider a new neighbourhood bank is being initiated and will set up its headquarters h, two of its branches b_1 and b_2, and four of its ATMs a_1, a_2, a_3, and a_4. They must construct a computer network such that the headquarters, branches, and ATMs can all communicate with each other. Additionally, they will need to network with the Federal Bank, f. To find its FFMST is being explained. The costs of the feasible network connections are listed below using existing score function and arithmetic operation of FFN is tabulated in Table 3. The illustration of the above problem is given in Figure 1 as a directed FFG $\mathcal{G}_{\mathcal{J}}$. Determining all the possible graphs $\mathcal{T}_{\mathcal{F}}$, the computed arborescence rooted at h of minimum cost is given in Figure 2.

Table 3. Costs of the feasible network connections

S.No.	Edges	Weights	Score function
1	hf	(0.8, 0.4)	0.724
2	hb_1	(0.4, 0.6)	0.424
3	hb_2	(0.8, 0.6)	0.648
4	b_1b_2	(0.7, 0.2)	0.6675
5	fb_1	(0.9, 0.3)	0.851
6	fa_1	(0.6, 0.7)	0.4365
7	b_1a_1	(0.5, 0.8)	0.3065
8	a_1a_2	(0.8, 0.3)	0.7425
9	ha_2	(0.7, 0.5)	0.609
10	b_2a_2	(0.9, 0.6)	0.7565
11	b_2a_3	(0.7, 0.3)	0.658
12	a_1a_4	(0.5, 0.3)	0.549
13	a_3a_4	(0.4, 0.5)	0.4695

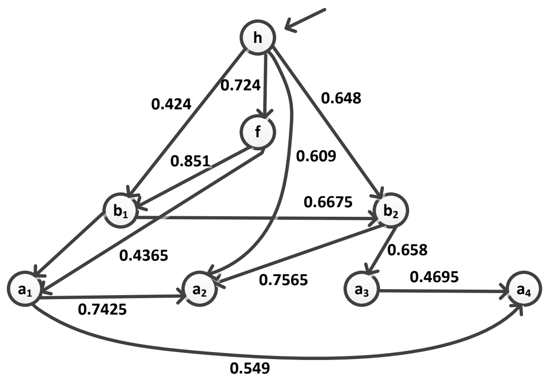

Figure 1. Graphical representation of example 1

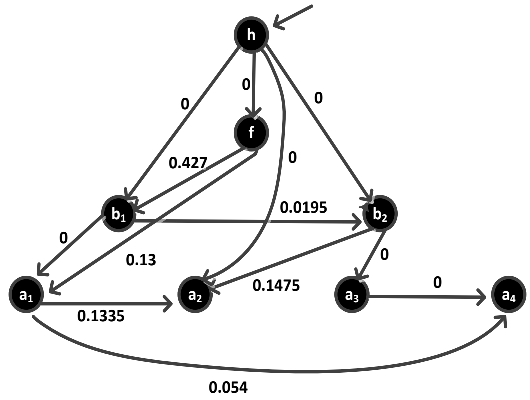

Figure 2. Computed arborescence

4.2. Result

By finding the $\mathcal{T}_{\mathcal{F}}$ FFMST for the above directed graph $\mathcal{G}_{\mathcal{J}}$ given in Figure 3 the minimum cost is found to be 3.839 by modified optimum branching algorithm. This is the estimated minimum cost for constructing a computer network such that the headquarters, branches, and ATMs can all communicate with each other.

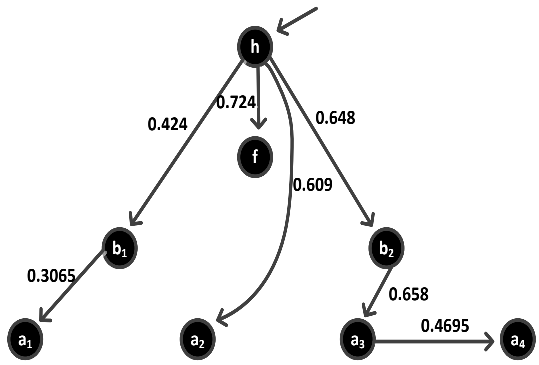

Figure 3. Resulted $\mathcal{T}_{\mathcal{F}}$, FFMST for the directed graph

4.3. Example 2

To determine the FFMST for the below given below by the proposed modified Prim's algorithm. Consider a connected undirected graph to findits FFMST. The existing score function and arithmetic operation of FFN for the given undirected graph is tabulated in Table 4. The illustration of the above problem is given in Figure 4 as an undirected FFG $\mathcal{G}_{\mathcal{J}}$.

Table 4. Costs of the feasible network connections

S. No.	Edges	Weights	Score function
1	ab	(0.4, 0.8)	0.276
2	ah	(0.8, 0.4)	0.724
3	bh	(0.8, 0.6)	0.648
4	bc	(0.4, 0.6)	0.424
5	cd	(0.6, 0.8)	0.352
6	ci	(0.6, 0.4)	0.576
7	cf	(0.7, 0.3)	0.658
8	df	(0.5, 0.3)	0.549
9	de	(0.4, 0.5)	0.4695
10	ef	(0.6, 0.7)	0.4365
11	fg	(0.5, 0.8)	0.3065
12	gi	(0.7, 0.2)	0.6675
13	gh	(0.8, 0.3)	0.7425
14	hi	(0.9, 0.3)	0.851

4.4. Result

By finding the $\mathcal{T_F}$FFMST for the above undirected graph $\mathcal{G_J}$ given in Figure 5 the minimum cost is found to be 3.8045 by modified Prim's algorithm. Above discussed real life applications demonstrates the effectiveness of the proposed algorithms in determining the minimum cost of the respective directed and undirected graphs involving FFN and its ranking technique.

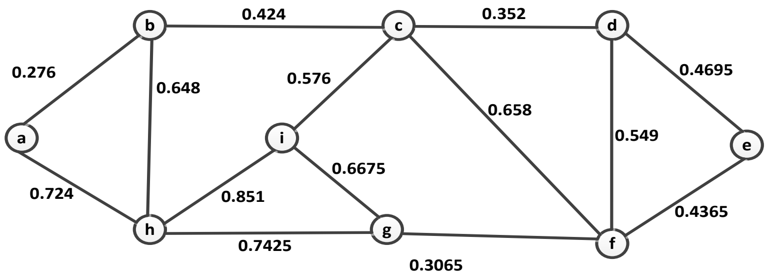

Figure 4. Graphical representation of example

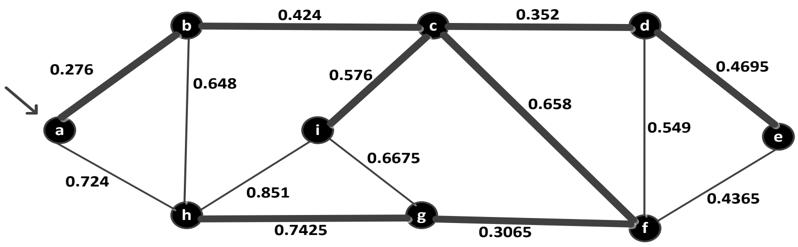

Figure 5. Resulted $\mathcal{T_F}$, FFMST for the undirected graph

5. CONCLUSION AND FUTURE WORK

In this work we have introduced algorithmic approach for solving FFMST problem with directed and undirected connected weighted FFG. A modified optimum branching algorithm for directed FFG and a modified Prim's algorithm for undirected FFG are proposed using the existing score function and arithmetic operation of FFNs. We have demonstrated the proposed algorithms using practical examples. In future the proposed methodology can be executed for a large-scale network problem. Further, the proposed algorithms can be extended for multi objective FFMST giving scope in forming most appropriate results for various problems.

REFERENCES

[1] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*. United States of America: Macmillan Education UK, 1976, doi: 10.1007/978-1-349-03521-2.

[2] R. C. Prim, "Shortest Connection Networks And Some Generalizations," *Bell System Technical Journal*, vol. 36, no. 6, pp. 1389–1401, Nov. 1957, doi: 10.1002/j.1538-7305.1957.tb01515.x.

[3] J. B. Kruskal, "On the shortest spanning subtree of a graph and the traveling salesman problem," *Proceedings of the American Mathematical Society*, vol. 7, no. 1, pp. 48–50, 1956, doi: 10.1090/s0002-9939-1956-0078686-7.

[4] R. L. Graham and P. Hell, "On the History of the Minimum Spanning Tree Problem," *IEEE Annals of the History of Computing*, vol. 7, no. 1, pp. 43–57, 1985, doi: 10.1109/mahc.1985.10011.

[5] H. Ishii, S. Shiode, T. Nishida, and Y. Namasuya, "Stochastic spanning tree problem," *Discrete Applied Mathematics*, vol. 3, no. 4, pp. 263–273, Nov. 1981, doi: 10.1016/0166-218x(81)90004-4.

[6] I. B. Mohd, "Interval elimination method for stochastic spanning tree problem," *Applied Mathematics and Computation*, vol. 66, no. 2–3, pp. 325–341, Dec. 1994, doi: 10.1016/0096-3003(94)90126-0.

[7] H. Ishii and T. Matsutomi, "Confidence regional method of stochastic spanning tree problem," *Mathematical and Computer Modelling*, vol. 22, no. 10–12, pp. 77–82, Nov. 1995, doi: 10.1016/0895-7177(95)00183-3.

[8] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965, doi: 10.1016/s0019-9958(65)90241-x.

[9] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, Aug. 1986, doi: 10.1016/s0165-0114(86)80034-3.

[10] R. R. Yager, "Pythagorean fuzzy subsets," in *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, Jun. 2013, pp. 57–61, doi: 10.1109/ifsa-nafips.2013.6608375.

[11] R. R. Yager, "Pythagorean Membership Grades in Multicriteria Decision Making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958–965, Aug. 2014, doi: 10.1109/tfuzz.2013.2278989.

[12] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 2, pp. 663–674, Jul. 2019, doi: 10.1007/s12652-019-01377-0.
[13] J. N. Mordeson and P. S. Nair, "Fuzzy Graphs," in *Fuzzy Graphs and Fuzzy Hypergraphs*, J. N. Mordeson and P. S. Nair, Eds. Heidelberg: Physica-Verlag HD, 2000, pp. 19–81, doi: 10.1007/978-3-7908-1854-3_2.
[14] P. Bhattacharya, "Some remarks on fuzzy graphs," *Pattern Recognition Letters*, vol. 6, no. 5, pp. 297–302, Dec. 1987, doi: 10.1016/0167-8655(87)90012-2.
[15] A. Shannon and K. Atanassov, "A first step to a theory of the intuitionistic fuzzy graphs," *Proceedings of the First Workshop on Fuzzy Based Expert Systems D. Lakov, ed.*, pp. 59–61, 2006.
[16] M. Akram and R. Akmal, "Intuitionistic fuzzy graph structures," *Kragujevac Journal of Mathematics*, vol. 41, no. 2, pp. 219–237, 2017, doi: 10.5937/kgjmath1702219a.
[17] M. Akram, A. Habib, F. Ilyas, and J. Dar, "Specific Types of Pythagorean Fuzzy Graphs and Application to Decision-Making," *Mathematical and Computational Applications*, vol. 23, no. 3, p. 42, Aug. 2018, doi: 10.3390/mca23030042.
[18] T. Itoh and H. Ishii, "An Approach Based on Necessity Measure to the Fuzzy Spanning Tree Problems," *Journal of the Operations Research Society of Japan*, vol. 39, no. 2, pp. 247–257, 1996, doi: 10.15807/jorsj.39.247.
[19] M. T. Takahashi and A. Yamakami, *A study of minimum spanning tree with fuzzy parameters–XXXV*. Natal, Brazil: Operational Research Brazilian Symposium (SBPO), 2003.
[20] T. A. de Almeida, A. Yamakami, and M. T. Takahashi, "An Evolutionary Approach to Solve Minimum Spanning Tree Problem with Fuzzy Parameters," in *International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06)*, doi: 10.1109/cimca.2005.1631469.
[21] A. Janiak and A. Kasperski, "The minimum spanning tree problem with fuzzy costs," *Fuzzy Optimization and Decision Making*, vol. 7, no. 2, pp. 105–118, Apr. 2008, doi: 10.1007/s10700-008-9030-5.
[22] M. Verma and K. K. Shukla, "Fuzzy Minimum Spanning Tree Problem: A Greedy Algorithm using Quasi-Gaussian Fuzzy Weights," *Journal of Software Engineering Tools & Technology Trends*, vol. 1, no. 3, Dec. 2014.
[23] A. Dey and A. Pal, "Prim's algorithm for solving minimum spanning tree problem in fuzzy environment," *Annals of Fuzzy Mathematics and Informatics*, vol. 12, pp. 419–430, Sep. 2016.
[24] J. Zhou, L. Chen, K. Wang, and F. Yang, "Fuzzy α-minimum spanning tree problem: definition and solutions," *International Journal of General Systems*, vol. 45, no. 3, pp. 311–335, Apr. 2016, doi: 10.1080/03081079.2015.1086578.
[25] A. Dey, S. Mondal, and T. Pal, "Robust and minimum spanning tree in fuzzy environment," *International Journal of Computing Science and Mathematics*, vol. 10, no. 5, pp. 513–524, 2019, doi: 10.1504/ijcsm.2019.103679.
[26] A. A. Deshpande and O. K. Chaudhari, "Fuzzy Approach to Compare a Minimal Spanning Tree Problem by Using Various Algorithms," *Advances in Fuzzy Mathematics*, vol. 15, no. 1, pp. 47–58, 2020.
[27] J. Gao and M. Lu, "Fuzzy quadratic minimum spanning tree problem," *Applied Mathematics and Computation*, vol. 164, no. 3, pp. 773–788, May 2005, doi: 10.1016/j.amc.2004.06.051.
[28] S. Majumder, S. Kar, and T. Pal, "Rough-fuzzy quadratic minimum spanning tree problem," *Expert Systems*, vol. 36, no. 2, p. e12364, Jan. 2019, doi: 10.1111/exsy.12364.
[29] K. Mohanta, A. Dey, N. C. Debnath, and A. Pal, "An algorithmic approach for finding minimum spanning tree in a intuitionistic fuzzy graph," in *EPiC Series in Computing*, doi: 10.29007/sx3s.
[30] A. Dey, L. H. Son, A. Pal, and H. V. Long, "Fuzzy minimum spanning tree with interval type 2 fuzzy arc length: formulation and a new genetic algorithm," *Soft Computing*, vol. 24, no. 6, pp. 3963–3974, Jun. 2019, doi: 10.1007/s00500-019-04166-1.
[31] S. Dan, S. Majumder, M. B. Kar, and S. Kar, "On type-2 fuzzy weighted minimum spanning tree," *Soft Computing*, vol. 25, no. 23, pp. 14873–14892, Sep. 2021, doi: 10.1007/s00500-021-06052-1.
[32] A. Habib, M. Akram, and C. Kahraman, "Minimum spanning tree hierarchical clustering algorithm: A new Pythagorean fuzzy similarity measure for the analysis of functional brain networks," *Expert Systems with Applications*, vol. 201, p. 117016, Sep. 2022, doi: 10.1016/j.eswa.2022.117016.
[33] L. Sahoo, "A new score function based Fermatean fuzzy transportation problem," *Results in Control and Optimization*, vol. 4, p. 100040, Sep. 2021, doi: 10.1016/j.rico.2021.100040.
[34] L. Sahoo, "Some Score Functions on Fermatean Fuzzy Sets and Its Application to Bride Selection Based on TOPSIS Method," *International Journal of Fuzzy System Applications*, vol. 10, no. 3, pp. 18–29, Jul. 2021, doi: 10.4018/ijfsa.2021070102.
[35] M. Blue, B. Bush, and J. Puckett, "Unified approach to fuzzy graph problems," *Fuzzy Sets and Systems*, vol. 125, no. 3, pp. 355–368, Feb. 2002, doi: 10.1016/s0165-0114(01)00011-2.

BIOGRAPHIES OF AUTHORS

Francis Remigius Perpetua Mary received her Ph. D. (2021) degree from SRM Institute of Science and Technology, M.Sc degree (2010) from Bharathidasan University, Tiruchirapalli, India and M.Phil degree (2011) in Mathematics from Madras University, Chennai, India. She is working as Assistant Professor, Department of Mathematics, College of Engineering and Technology SRM Institute of Science and Technology, India. Her researches of interest are in the area of applied mathematics which includes graph theory and its application. She can be contacted at email: maryf@srmist.edu.in.

Swaminathan Mohanaselvi is currently working as Assistant Professor in the Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur, Chennai, India. She has been awarded Ph. D degree from SRM University. She has more than 18 years of experience in teaching. Her research interests include fuzzy optimization and uncertainty theory. She can be contacted at email: mohanass@srmist.edu.in.

Dr. Broumi Said completed his MSc and PhD in Computer Science from University of Hassan II of Casablanca, Morocco. Currently, he is serving as an Assistant Professor at Regional Center for the Professions of Education and Training, Casablanca-Settat, Morocco. His research interests are in the field of graph theory, extended fuzzy graphs, decision analysis and neutrosophic theory. He has diversity in his research work. He has published more than 200 research articles in international peer-reviewed ISI indexed/impact factor journals. He can be contacted at email: broumisaid78@gmail.com.